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RESEARCH ARTICLE

Strategic complementarity of information in financial markets with large shocks

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Abstract In a simple model of a frictionless financial market with rational agents, the value of private information increases when large discrete shocks independently affect the fundamental value of the asset and the exogenous trading. The complementarity in information gathering generates multiple equilibria.

Keywords Endogenous information · Strategic complementarity · Financial markets · Aggregation of information

JEL Classification D82 · G12 · G14

1 Introduction

In the crash of October 1987, no sudden news about fundamentals could be found that would have justified the quantum fall of the market price. An exogenous shock of demand has been mentioned as one of the causes.[1] Trading strategies driven by portfolio insurance may have amplified an initial exogenous disturbance and generated a quantum jump in exogenous sales. In this context, can the value of information about the fundamentals increase with the number of informed agents? An argument for a positive answer is presented here.

Private information about the fundamental value of an asset is valuable because it augments the profit from trading that arises from the gap between the agent's expected value of the asset and its market price. If more agents are informed, the market price

[1] See among others, Genotte and Leland (1990).

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should reflect that information, thus reducing the gap between private information and public price, and the value of private information. That property is shown by Grossman and Stiglitz (1980) in the special case of the CARA-Gauss model where all random variables are Gaussian and agents have constant absolute risk-aversion. In that model, both the trade of the informed agents and the noise have a Gaussian distribution. A larger mass of informed agents increases the signal to noise ratio in the observed order flow. The market price is then a better signal of the fundamental and the value of information decreases. A similar mechanism may operate for a wide class of single-peaked distributions of beliefs and of noise trading. In this case, when there are more informed agents, large volumes of observed trades tend to be attributed more to informed agents and the market is more informative.

When quantum jumps may independently affect the fundamental or the noise trade, the Gaussian assumption is not appropriate. The possibility of such jumps is the essential assumption here, under which an increase of the mass of informed agents may increase the value of information.

The mechanism has a simple description. When there are few informed agents, trades of moderate size provide information while large trades are dismissed as being driven by a noise shock. In this case, a higher number of informed agent increases the signal to noise ratio and lowers the value of private information. However, if the number of informed agents keeps rising, then a large trade may be driven by agents informed about a quantum jump in the value of the asset. Non-informed agents cannot know from the market whether the large trade is due to a shock on the fundamental or on the noise trade. The signal to noise ratio is smaller than when the mass of informed agents is relatively small, and the value of private information is higher.

The issue is particularly relevant if private information is endogenous and can be obtained at some cost. If the value of information for an informed agent increases with the information of other agents, the strategic complementarity may generate multiple equilibria and sudden switches between market regimes of low information gathering and trading volume, and regimes of frenzies in information and trading.

The model that is presented here departs from the standard model of Grossman and Stiglitz on the distribution assumptions of the fundamental and of the noise. A related model is that of Froot et al. (1992) where the value of the private signal increases with the number of agents owning the signal, but there is a friction in the market that causes the price to not fully reflect the information generated by trades. In Chamley (2007), the value of information increases with the mass of informed agents in a model without friction when agents trade for short-term gains, i.e. undo their position before the revelation of the fundamental. A key assumption there, as in the present paper, is that some news may increase the uncertainty in the public information about the fundamental, a property that cannot hold in the Gaussian framework.

2 The model and the equilibrium

Following a standard micro-structure of a financial market, a financial claim to a fundamental is traded in a one-period market with three types of agents: informed agents, market-makers with no private information and noise traders.

In order to highlight the role of large shocks, the fundamental value of the asset, θ, takes one of two values: $\theta \in \{\theta_0, \theta_1\}$, with $\theta_0 < \theta_1$. (Other distributions could be considered and a Gaussian variable of moderate variance could be added to the payoff). Without loss of generality, the two values are normalized such that $\theta_0 = 0$ and $\theta_1 = 1$, but the notation $\{\theta_0, \theta_1\}$ will also be used when convenient. The high state $\theta = \theta_1$ has a prior probability μ_θ.

There is a continuum of mass I of informed agents who know the value of θ. They are risk-neutral but constrained in their trade. The trade constraints are normalized such that an agent cannot trade more than one unit of the asset, in absolute value.[2] Hence, each agent trades $2\theta - 1$, and the demand by informed agents is $I(2\theta - 1)$.

The exogenous demand is equal to $\zeta + \eta$ where $\zeta \in \{\zeta_0, \zeta_1\}$ and η is independent, $\eta \sim \mathcal{N}(0, \sigma^2)$. We normalize $\zeta_0 = 0$ and $\zeta_1 = 1$. Let μ_ζ be the probability that $\zeta = \zeta_1 = 1$. There is no restriction on the probabilities of the shocks to the fundamental, but a particularly relevant case is that of probabilities μ_θ and μ_ζ that are close to 0 or 1, where one of the states defines a quantum jump.

Finally, there is a continuum of uninformed agents who are rational. In order to simplify the solution of the model, these agents are risk-neutral and not constrained in their trade. In this setting, the expected value of the fundamental is equal to the equilibrium price.

2.1 The market equilibrium

As is well known, the information conveyed by the market equilibrium is equivalent to the observation of the order flow which is the sum of the demands of the informed and the noise traders. In the state (θ, ζ), this order flow is equal to

$$y(\theta, \zeta, \eta) = I(2\theta - 1) + \zeta + \eta. \tag{1}$$

The observation of the order flow y is informationally equivalent to the observation of the variable

$$Y = 2I\theta + \zeta + \eta. \tag{2}$$

The market conveys a signal about θ that is garbled by the additive noise term $\zeta + \eta$. Equation (2) is a fundamental equation in this paper as it summarizes the information conveyed by the market equilibrium.

Because of the risk-neutral uninformed agents, the equilibrium price is such that

$$p = E[\theta|Y] = P(\theta = \theta_1|Y)(\theta_1 - \theta_0) + \theta_0. \tag{3}$$

Let $\pi(\theta, \zeta)$ be the prior probability of the realization (θ, ζ), and $\mu(\theta, \zeta|Y)$ the probability (belief) after the observation of the equilibrium price. Since η has a Gaussian distribution $\mathcal{N}(0, \sigma^2)$, the post-equilibrium belief can be written

[2] One could also consider a constraint on the monetary value of the transaction.

$$\mu(\theta, \zeta | Y; I) = A \exp\left(-\frac{1}{2\sigma^2}(Y - 2I\theta - \zeta)^2\right)\pi(\theta, \zeta), \tag{4}$$

where the constant A is such that the sum of the probabilities is equal to one, and Y is determined by (2).

Substituting Y in (4) by its expression in (2), the posterior distribution over (θ, ζ) is a function of the true state $(\hat{\theta}, \hat{\zeta})$, the noise η, and the mass of informed agents I such that

$$\mu(\theta, \zeta | \hat{\theta}, \hat{\zeta}, \eta; I) = A \exp\left(-\frac{1}{2\sigma^2}\left(2I(\hat{\theta} - \theta) + \hat{\zeta} - \zeta + \eta\right)^2\right)\pi(\theta, \zeta). \tag{5}$$

From (3), using here $\theta_0 = 0$, $\theta_1 = 1$, the equilibrium price of the asset depends on the true values of the fundamental, the two components of the demand shock and the mass of informed traders:

$$p(\hat{\theta}, \hat{\zeta}, \eta; I) = \mu(\theta_1, \zeta_1 | \hat{\theta}, \hat{\zeta}, \eta; I) + \mu(\theta_1, \zeta_0 | \hat{\theta}, \hat{\zeta}, \eta; I). \tag{6}$$

2.2 The value of information

Before the market opens, the agent knows that two possibilities may arise.

- If he learns that the true state is $\hat{\theta} = \theta_1$ (with the prior probability μ_θ), then he will buy one unit of the asset (up to his constraint), and get a profit of $1 - p(\hat{\theta}_1, \hat{\zeta}, \eta; I)$. The price $p(\hat{\theta}_1, \hat{\zeta}, \eta; I)$ depends on the independent random variables $\hat{\zeta}$ and η. The first is binary and equal to ζ_1 with probability μ_ζ, and η has the density $\phi(\eta)$. Taking the expectation over all possible realizations of (ζ, η) and using (6), the expected profit, contingent on learning that $\theta = \theta_1$, is equal to

$$B(I) = \int \left(1 - \mu_\zeta p(\theta_1, \zeta_1, \eta; I) - (1 - \mu_\zeta) p(\theta_1, \zeta_0, \eta; I)\right)\phi(\eta) d\eta.$$

- If the agent learns that the true state is $\hat{\theta} = \theta_0$, then he will sell one unit of the asset and get a profit equal to its price $p(\hat{\theta}_0, \hat{\zeta}, \eta; I)$ which depends on the random variables $\hat{\zeta}$ and η. As in the previous case, the agent computes the expectation of the profits for this stragegy.

Summing over the two possible events $\hat{\theta} = \hat{\theta}_1$, $\hat{\theta} = \hat{\theta}_0$, with probabilities μ_θ and $1 - \mu_\theta$ respectively, the value of information is a function of I that is defined by

$$\begin{aligned} V(I) = \mu_\theta &\int \left(1 - \mu_\zeta p(\theta_1, \zeta_1, \eta; I) - (1 - \mu_\zeta) p(\theta_1, \zeta_0, \eta; I)\right)\phi(\eta) d\eta \\ &+ (1 - \mu_\theta) \int \left(\mu_\zeta p(\theta_0, \zeta_1, \eta; I) + (1 - \mu_\zeta) p(\theta_0, \zeta_0, \eta; I)\right)\phi(\eta) d\eta. \end{aligned}$$

3 Properties

We consider two cases: In the first, the mass of informed agents is relatively small; in the second, this mass is such that trading by informed agents is commensurate with exogenous jumps of the demand.

Assume first that the mass of informed agents has an "intermediate value", $I_1 = 1/4$. The difference between the order flow of informed agents in the low and the high state is equal to $1/2$, which is half of the difference between the mean of the distribution of the demand with and without shock, respectively, [see Eq. (2)]. If the order flow jump is driven by information about the fundamental value, it is equal to 1/2, whereas if it is driven by a demand shock, it is equal to 1. When the variance of the noise term η is small, the two shocks which are independent, can be (albeit not perfectly) separated by the order flow in equilibrium. The market price is on average close to the fundamental value. That is the meaning of the next result.

Proposition 1 If $I = I_1 = 1/4$, the price is close to the fundamental in the following sense: for any $\alpha > 0$, $\delta > 0$, there exists σ^* such that if the variance σ^2 of the noise trade satisfies $\sigma < \sigma^*$,

$$P(|p(\theta, \zeta, \eta; I_1) - \theta| > \alpha) < \delta,$$

where the probability P is measured with the prior distribution on (θ, ζ, η).

Proposition 1 shows that when the variance of the noise term η is sufficiently small, the price is arbitrarily close to the fundamental with a probability arbitrarily close to one. Since the payoff of information rests on the difference between the price and the fundamental, the value of information must be arbitrarily small. From Proposition 1, and since the gain from trade is bounded by 1, it follows immediately that $V(1/4) < \delta + (1 - \delta)\alpha$. Changing the notation about α, we have the next result.

Proposition 2 For any $\alpha > 0$, there exists σ^* such that if $\sigma < \sigma^*$, $V(I_1) < \alpha$.

We now consider the case where the mass of informed agents is $I_2 = 2I_1 = 1/2$. The observation of the market is equivalent to the observation of the variable Y in (2) which becomes now

$$Y' = \theta + \zeta + \eta. \tag{7}$$

The states $(\theta = 1, \zeta = 0)$ and $(\theta = 0, \zeta = 1)$ have the same impact on Y' and cannot be separated by the observation of Y'. In these two states, the market price remains bounded away from 0 or 1 when the variance σ^2 of the noise η becomes arbitrarily small. The value of information is bounded away from zero. This argument is formalized in the next result.

Proposition 3

$$V(I_2) > \frac{2\mu_\theta \mu_\zeta (1 - \mu_\theta)(1 - \mu_\zeta)}{\mu_\theta (1 - \mu_\zeta) + (1 - \mu_\theta)\mu_\zeta}.$$

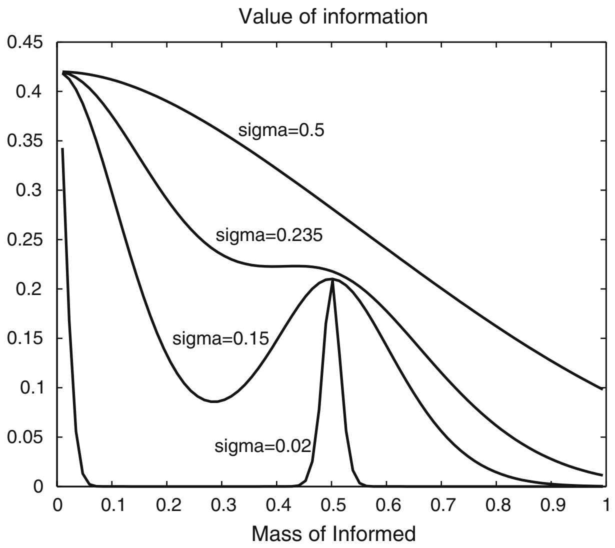

Fig. 1 The value of information as a function of the mass of informed agents

From the previous propositions, we have immediately the next result.

Theorem 1 *Let $V(I)$ be the value of information (before the market opens) as a function of the mass I of informed agents. There exists $\bar{\sigma}$ such that if the variance of the exogenous demand σ^2 is smaller than $\bar{\sigma}^2$, then $V(1/4) < V(1/2)$, and the value of information is increasing in some interval contained in* $(0, 1/2)$.

3.1 An example

The function $V(I)$ is computed numerically and represented for four values of the variance parameter σ in Fig. 1. Propositions 1 and 2 are illustrated by the cases $\sigma = 0.02$ and $\sigma = 0.15$: when I is in some range between 0 and $1/2$, the order flow provides very good information on the fundamental and the value of information is very small, not distinguishable from 0 if $\sigma = 0.02$. Following Proposition 3 and given the parameters of the example, for all values of σ, $V(1/2) > 0.21$. When $I = 0$, the market provides no information, and information has its maximum value. When I increases from 0, the market provides some information, and the value of private information must be decreasing from its maximum. If σ is sufficiently small, the *ex ante* value of information may decrease to a very low level. But if I increases toward $1/2$, the market provides less information when only one shock occurs and the value of information increases. In the region $I > 1/2$, the case is similar to the one for small I, and $V(I)$ is decreasing in I.

3.2 Endogenous information acquisition

So far, the mass I of informed agents has been exogenous. Assume now that there is a large mass of agents, trade-constrained as defined previously, who can buy the signal $s = \theta$ at the fixed cost c. (One could also have an heterogenous population with different fixed costs). The supply of informed agents is perfectly elastic at the cost c. Assume that $V(1/4) < c < V(1/2)$. In this case, there are at least three values of I such that $V(I) = c$. At least two of these values define a stable equilibrium.

The next proposition follows immediately from the previous results.

Proposition 4 Assume that the supply of informed agents (who are risk-neutral and trade-constrained) is endogenous and infinitely elastic at the cost c. For any given parameters of the model, except σ, there is a value $\bar{c}$, such that if $c < \bar{c}$, there exists $\underline{\sigma}$ such that if $\sigma < \underline{\sigma}$, there are at least two stable equilibrium values of the mass of endogenous agents.

4 Conclusion

The results of the paper were established for normalized values of jumps. The extension to arbitrary values of jumps is a trivial exercise. The main assumption for the present argument is the possibility of independent quantum jumps in both the fundamental and the exogenous demand. When jumps are restricted to either the fundamental or the exogenous demand, the standard property of a decreasing value of information seems to hold. It remains to be seen if other mechanisms could generate such a strategic complementarity when agents trade in a frictionless market for the long-term and hold until the revelation of the fundamental. As shown in Chamley (2007) in a model with a Gaussian distribution of noise, strategic complementarity in information acquisition arises when agents hold their position for a short time and trade for short-term profit.

The existence of multiple equilibrium levels of information gathering raises the possibility of large price jumps in the context of multiple trading rounds. In the example of Fig. 1, the probabilities of shocks, either for the fundamental or the exogenous demand, is small at 1 percent. Suppose now that the normal state is $\theta = 1$, that a shock occurs with $\theta = 0$, and that information gathering is at a low information equilibrium. When there are multiple trading rounds, the expected value of the fundamental is initially near 1, but it must eventually converge since it is a bounded martingale, and it converges to the true value, 0. When the public belief of the good state μ_θ decreases, there is more uncertainty about the fundamental. This higher uncertainty may increase the value of gathering information such that the low equilibrium disappears in some round. The mass of informed agents may jump up in which case the market becomes suddenly highly informative. While there is no exogenous news, the price may suddenly crash.

Proofs

Proposition 1 To simplify the notation, we omit the variable I (which is equal to I_1) from the arguments of μ and p in their expressions (5) and (6). If $(\hat{\theta}, \hat{\zeta})$ is the true state of the economy, the posterior belief takes the form

$$\begin{aligned}\mu(\theta,\zeta|\hat{\theta},\hat{\zeta},\eta) =& A\exp\left(-\frac{1}{2\sigma^2}\left(z(\theta,\zeta;\hat{\theta},\hat{\zeta})+\eta\right)^2\right)\pi(\theta,\zeta), \quad \text{with}\\ z(\theta,\zeta;\hat{\theta},\hat{\zeta}) =& \frac{\hat{\theta}-\theta}{2}+\hat{\zeta}-\zeta.\end{aligned} \tag{8}$$

For any $(\theta,\zeta)\neq(\hat{\theta},\hat{\zeta})$, and since $z(\hat{\theta},\hat{\zeta};\hat{\theta},\hat{\zeta})=0$,

$$\frac{\mu(\theta,\zeta|\hat{\theta},\hat{\zeta},\eta)}{\mu(\hat{\theta},\hat{\zeta}|\hat{\theta};\hat{\zeta},\eta)}=\exp\left(-\frac{z(\theta,\zeta;\hat{\theta},\hat{\zeta})}{\sigma^2}\left(\frac{z(\theta,\zeta;\hat{\theta},\hat{\zeta})}{2}+\eta\right)\right)\frac{\pi(\theta,\zeta)}{\pi(\hat{\theta},\hat{\zeta})}. \tag{9}$$

Since η has a distribution $\mathcal{N}(0,\sigma^2)$, for any $\delta>0$, there exists $K>0$ such that $P(|\eta/\sigma|>K)<\delta$.

There exists $M>0$ such that $|z(\theta,\zeta;\hat{\theta},\hat{\zeta})|>M$ for all $(\theta,\zeta)\neq(\hat{\theta},\hat{\zeta})$, because there is a finite number of these values and they are all different from zero.

If follows that with probability at least $1-\delta$,

$$\frac{z(\theta,\zeta;\hat{\theta},\hat{\zeta})}{\sigma}\left(\frac{z(\theta,\zeta;\hat{\theta},\hat{\zeta})}{2\sigma}+\frac{\eta}{\sigma}\right)>\frac{M}{\sigma}\left(\frac{M}{2\sigma}-K\right).$$

Using (9), for any $\gamma>0$ there exists σ^* such that if $\sigma<\sigma^*$, for any $(\theta,\zeta)\neq(\hat{\theta},\hat{\zeta})$,

$$\frac{\mu(\theta,\zeta|\hat{\theta},\hat{\zeta},\eta)}{\mu(\hat{\theta},\hat{\zeta}|\hat{\theta},\hat{\zeta},\eta)}<\gamma.$$

Because the sum of the probabilities of all (θ,ζ) is equal to one,

$$\mu(\theta,\zeta|\hat{\theta},\hat{\zeta},\eta)<\gamma, \quad \text{and} \quad \mu(\hat{\theta},\hat{\zeta}|\hat{\theta},\hat{\zeta},\eta)>1-3\gamma. \tag{10}$$

Since the price is given by $p(\hat{\theta},\hat{\zeta},\eta)=\mu(\theta_1,\zeta_0|\hat{\theta},\hat{\zeta},\eta)+\mu(\theta_1,\zeta_1|\hat{\theta},\hat{\zeta},\eta)$, using (10),

$$P\left(|p-\hat{\theta}|<4\gamma\right)>1-\delta.$$

The proof is concluded by choosing $\gamma=\alpha/4$, where α is the parameter in the proposition.

Proposition 3 The profit from trading by a non-informed agent is based on the observation of the variable y given in (1), which, when $I=1/2$, is equivalent to $Y=y+0.5=\hat{\theta}+\hat{\zeta}+\eta$. His (maximum) expected profit is zero. The signal y' is less informative than the signal $\hat{\theta}+\hat{\zeta}$ in the sense of Blackwell (1951). If an agent could observe the signal $\hat{\theta}+\hat{\zeta}$ without the noise η, and trade at the equilibrium price (6), his expected profit would be strictly positive. The value of information is the difference between the expected profit from trade by an informed agent and the non-informed agent. Therefore, this value is bounded below by the difference between the expected trade profit of an informed agent and an agent who observes $\hat{\theta}+\hat{\zeta}$.

The agent who observes $\hat{\theta} + \hat{\zeta}$ has perfect information in the states (1, 1) and (0, 0). Hence, we can restrict the comparison with the payoff of the perfectly informed agent to the states with $\hat{\theta} \neq \hat{\zeta}$.

$$V\left(\frac{1}{2}\right) > \mu_\theta(1 - \mu_z)\left(1 - P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1)\right) + (1 - \mu_\theta)\mu_\zeta P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1).$$

By application of Bayes' rule,

$$P(\hat{\theta} = 1|\hat{\theta} + \hat{\zeta} = 1) = \frac{(1 - \mu_\zeta)\mu_\theta}{(1 - \mu_\zeta)\mu_\theta + \mu_\zeta(1 - \mu_\theta)}.$$

Substituting in the previous expression, one finds the inequality in Proposition 3.

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